Article

Vectorial Image Representation for Image Classification

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**Abstract:** This paper proposes the transformation , where **S** is a digital gray- level image and is a vector expressed through the textural space. The proposed transformation is denominated Vectorial Image Representation on the Texture Space (VIR-TS), given that the digital image **S** is represented by the textural vector . This vector contains all of the local texture characteristics of the image-of-interest, and the texture unit entertains a vectorial character, since it is defined through the resolution of a homogenous equation system. For the application of this transformation, a new classifier for multiple classes is proposed in the texture space, where the vector is employed as a characteristics vector. To verify its efficiency, it was experimentally deployed for the recognition of digital images of tree barks, obtaining an effective performance. In these experiments, the parametric value λ employed to solve the homogenous equation system does not affect the results of the image classification. The VIR-TS transform possesses potential applications in specific tasks, such as missing persons localization, and the analysis and classification of diagnostic and medical images.

**Keywords:** Vectorial Image Representation on the Texture Space (VIR-TS); Texture Unit ; Homogeneous Equation System; Multiclass Classifier; Digital Image Recognition.

1. Introduction

The visual texture is an important element for component classification in scenes and is commonly used for the processing of visual information. The surfaces of all materials are characterized through their texture properties, which can be described as follows: a) the visual texture is a spatial distribution of gray levels; b) the visual texture can be perceived through different scales or resolutions; c) the texture is an area property and not a point property; d) a region is perceived as texture when the number of primitive objects within it is large. On the other hand, according to reference [1], some important perceptions of the quality of a texture are the following: uniformity; density; rugosity; linearity; direction; frequency, and phase. Henceforth, a texture can be considered as fine, rough, soft, regular, irregular, or linear. The grade of irregularity or the properties of a texture can be found scattered throughout the entire image. In the field of texture analysis, there exist three major problems: a) texture classification, focused on determining to which class the sampled texture belongs [2-4]; b) texture segmentation, where an image is sectioned into multiple regions where each region has a specific type of texture [5,6]; and c) texture synthesis, which focuses on constructing a model that could be employed to produce artificial textures for specific applications such as computer graphics [7,8]. Furthermore, according to reference [9], the characteristics extraction techniques can be classified into four categories: geometrical; based on signal processing, and statistical models. The geometrical methods are based on the analysis of primitive textures. Some geometrical methods for primitive extractions include adaptative region extractions, mathematical morphology, structural methods, and border detection [10,11]. The model-based methods hypothesize the subjacent texture, constructing a parametric model that can generate the intensity’s distribution-of-interest. Ergo, these models can also be employed for texture synthesis. Some of these models that are applied for texture synthesis are called stochastic spatial interaction models, random field models, and fractals [12,13]. The signal processing methods perform an analysis of the frequency components of the images; the latter are also known as filtering methods, and to mention only some of these, we submit spatial domain filter, frequency analysis, and spatial/spatial-frequency methods [14,15]. Last but not least, the statistical methods offer an analysis of the spatial distribution of the local texture characteristics. Such characteristics are represented through a histogram of a variable dimension depending on the procedure employed to calculate the texture unit [16-18]. This histogram presents the occurrence frequency of the estimated texture units within the digital image, and its dimension is dependent on the unit texture definition. Selection of the texture extraction method is conducted in agreement with the problematic-at- hand. There are two types of classifiers for image classification in an *a priori* knowledge scheme: one-class and multiclass. For one-class classifiers [19,20], an unequivocal class is clearly defined, while the remaining classes are of no interest. In this situation, a region is defined within the characteristics space; this region represents the textural characteristics of the known class. This region is the acceptance zone for the class-of- interest or is employed as a prototype. On the other hand, in the multiclass classifiers [21-23], the characteristics space is divided into multiple regions, each region corresponding to the characteristics of a class and, frequently, the class (image) is represented by a characteristics vector known as a prototype vector. The classification of multiclass images consists of comparing the characteristics vector of a test image with the characteristics vectors of the known classes. Henceforth, the test image is assigned to the class with the most similar characters. This discrimination is performed by means of the distance between the vectors within the characteristics space.

To our knowledge, the texture unit has not been defined through a homogeneous equation system, which is defined through an observation window. In this paper, the local texture characteristics are extracted from grayscale images **.** To extract the texture characteristics, a mobile observation window of in size is employed to detect local random patterns of pixels across the image. In each detected position, the pixel values are considered constants within a homogenous equation system whose solution is the vectorial unit texture . This unit is represented in a new texture space as a vector radius that from the origin to the vector position , such that each random pattern of pixels have a corresponding texture unit vector (vector radius). By adding together all of the components of the vector radius, the is calculated; this latter vector contains all of the local texture characteristics of the image-under-study **S**. Ergo, the transformation represents a gray-level image S through a vector , whose direction and magnitude depend entirely on the textures of the image. This transformation has been denominated Vectorial Image Representation on the Texture Space (VIR-TS), due to the representation of a digital image **S** through the vector . The efficiency of the VIR-TS transform was experimentally corroborated through the classification of tree stems images with a multiclass classifier, where the vector is employed as a characteristics vector.

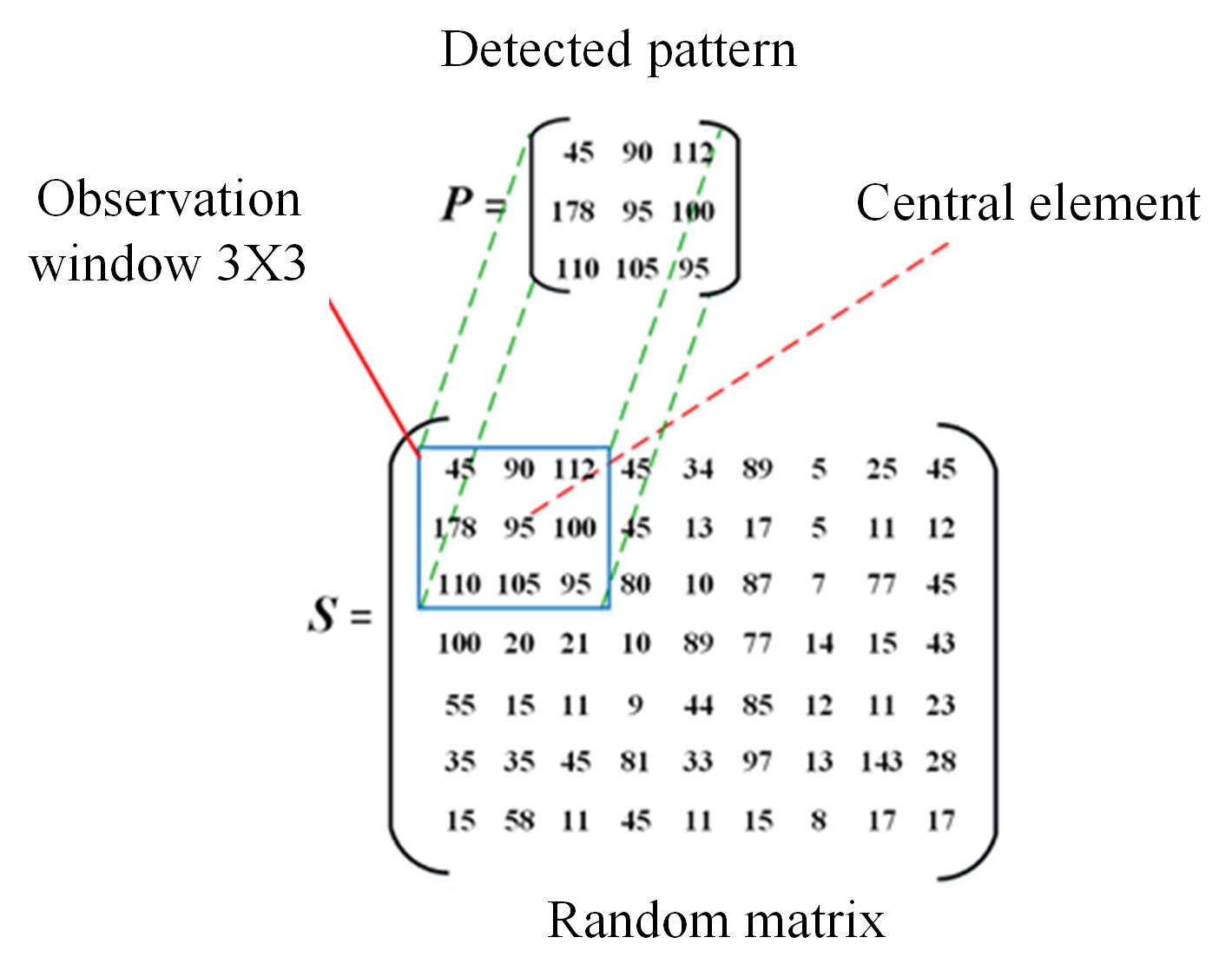
The work has the following structure. Materials and methods are presented in section 2. In section 2.1, the texture space is described based on three subsections: in 2.1.1 the definition of the texture unit is shown; in section 2.1.2, the definition of the texture unit is represented graphically and in section 2.1.3. describes the representation of a digital image in texture space. In section 2.2, the procedure to measure the similarity in texture space between a prototype vector and a test vector is explained. Section 2.3 describes a classifier for multiple classes in texture space and where the VIR-TS vector is used as a feature vector. In section 3 the experimental work is developed. In section 3.1, a digital image database is vector represented in texture space where each vector has its own direction and magnitude. Furthermore, using the vectors obtained in the transformation, the similarity between images is measured. In section 3.2, experimental results of image classification are reported where the high efficiency of the VIR-TS technique is demonstrated. A discussion of our work is described in section 4. Finally, in section 5 the most relevant conclusions are presented.

2. Materials and Methods

2.1. Texture space

2.1.1. Texture unit definition

In the texture analysis, a mobile observation window W frequently bears a size [21, 24, 25]; it is deployed to extract the local texture characteristics of an image- under-study. This window is shifted pixel-by-pixel across the whole image and, for each position, the window detects a discrete pattern, which is employed to generate a decimal code called a texture unit. Afterward, the texture unit is interpreted as a discrete variable and is then taken as an index to generate a discrete histogram . Such a histogram is interpreted as a texture spectrum and is then deployed as a characteristics vector in image classifiers [21].

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**Figure 1.** A pattern ***P*** detected in the grayscale image ***S*** through an observation window of 3×3 elements.

Now, bearing in mind the structure of the mobile observation window, and considering the gray-level image such as a random matrix , with an size, and for each position, a discrete pattern is detected through the window, as shown in Figure 1. If the pattern elements are considered the coefficient of a homogeneous equation system, the system will be:

|  |  |
| --- | --- |
|  | (1) |

Where is termed the coefficient matrix of the homogeneous linear system, represented as a matrix of real elements and will be called the unit texture vector. The trivial solution of the homogeneous equation system occurs when all of the elements of vector have a value of zero: , , . Nonetheless, this solution is not functional for our interests; thus, a non-trivial solution must be found. Therefore, based on a linear algebra concept, the non-trivial solution is possible when its determinant is equal to zero; as a consequence, there will be infinite solutions. To achieve this, the term is introduced within the equations and their determinant is equal to zero, as shown in Equation 2:

|  |  |
| --- | --- |
|  | (2) |

Hence, the problem becomes that of finding a *K* value, so that the condition is satisfied. From Equation (2), in terms of the matrix elements , *K* will have a value of

|  |  |
| --- | --- |
|  | (3) |

Once the value K has been determined, it is introduced into the equation system; Equation (1) then takes the following form:

|  |  |
| --- | --- |
|  | (4) |

Where the value is determined by Equation (3).

Afterward, to determine the texture unit , the non-trivial solution of Equation (4) must be found. As a first step, the first two linear equations are left depending on ,

|  |  |
| --- | --- |
|  | (5) |

employing the Cramer Rule method, the solution for is obtained through:

|  |  |
| --- | --- |
|  | (6) |

While the solution for will be:

|  |  |
| --- | --- |
|  | (7) |

Where D is the determinant of the 2×2 equation system, is the determinant for and is the determinant for . It is noteworthy that and function on the basis of ; accordingly, the infinite solution in parametric form would be:

|  |  |
| --- | --- |
|  | (8) |

Observing expression (8), for each real value of lambda , a unique resolution of the infinite solution is found. For example, when , the trivial solution of the equation system is obtained (, , and ); henceforth, the non-trivial solution is obtained when .

2.1.2. Graphical representation

Based on Equation (1) and Expression (8), the unit texture vector is defined by: It can be represented through the Cartesian coordinate-system form of:

|  |  |
| --- | --- |
|  | (9) |

Where **,** , are the unit vectors that indicate the axis direction in a rectangular coordinate system of three dimensions (Figure 2a). Hereafter (9), the scalars are the components of vector in the directions . Finally, from Equation (8), the magnitude of vector is:

|  |  |
| --- | --- |
|  | (10) |

And its directing cosines are:

|  |  |
| --- | --- |
|  | (11) |

Where:

|  |  |
| --- | --- |
|  | (12) |

With being the magnitude of vector ; its graphic presentation is displayed in Figure 2b. Based on Figure 2b, the texture unit is a radius vector goes from the origin to the coordinates .

It is clear that the direction and magnitude depend on the λ value and the elements of the pattern.

|  |  |
| --- | --- |
|  |  |

**Figure 2.** Representation of unit in the texture space: (a) graphic representation of unit vectors **,** ,; (b) graphic representation of texture unit and its components .

2.1.3. Image representation on the texture space

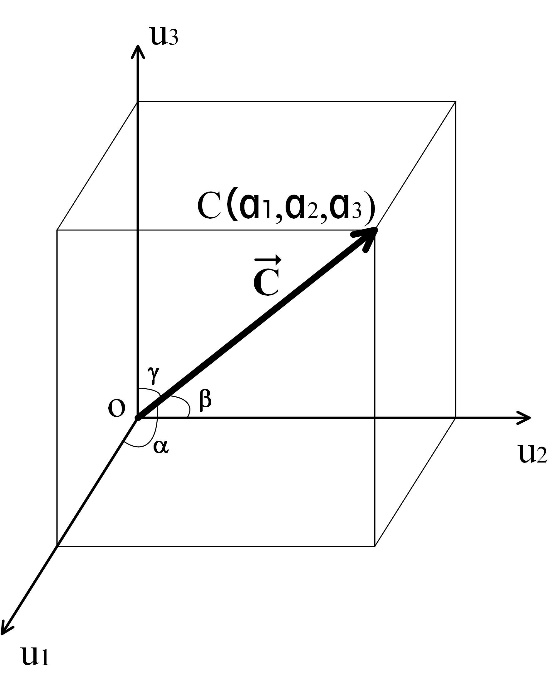
Given that, if a grayscale image S has an size and if this image is analyzed through an window, then there are patterns . Furthermore, given that each pattern (in the image domain) generates a texture unit (in texture space), then when the image is analyzed locally through the observation window for the pattern , the texture unit is calculated (radius vector in texture space); as a consequence, the image can be represented through a series of radius vectors. Thus, adding together all of the components of all of these radius vectors in the texture space, the image is represented with the vector , defined by:

|  |  |
| --- | --- |
|  | (13) |

The directions are given by , and the components are calculated with:

|  |  |
| --- | --- |
|  | (14) |

where is the component of the elements for , is the component of the elements for , and is the component of the elements for . Equation (9) was considered, and is the total of found patterns of the digital image-under- study. Figure 3 depicts the vector ,which is in texture space.



**Figure 3.** Graphic representation of the texture vector with its directing cosines.

Considering Figure 3 and Equation (13), the magnitude of vector will be:

|  |  |
| --- | --- |
|  | (15) |

Where its directing cosines are given with:

|  |  |
| --- | --- |
|  | (16) |

And holding the equivalence:

|  |  |
| --- | --- |
|  | (17) |

Based on the performed analysis, the image can be represented as a radius vector in the texture space whose magnitude and direction depend on the randomness of the image-under-study.

2.2. Similarity Measurement Between a Prototype Image and Test Image

With the knowledge that the transformation is possible, then the measurement of similarity between a prototype image and an unknown test image can be performed in the texture space.

Given a digital image of a c class whose texture vector is and given an unknown test image whose vector is , the difference between the and images in the texture space can be calculated through subtraction of the unknown image minus the vector of the prototype image :

|  |  |
| --- | --- |
|  | (18) |

Where is the difference vector between the texture images.



**Figure 4.** Geometry employed for the similarity measurement between the and.

Images, deploying the and vectors. Considering the geometry present in Figure 4 and the cosines law, we obtain:

|  |  |
| --- | --- |
|  | (19) |

And from (19), we would obtain:

|  |  |
| --- | --- |
|  | (20) |

Due to the geometry of the problem, if (18) is substituted in (20), we obtain:

|  |  |
| --- | --- |
|  | (21) |

On applying the distributive law:

|  |  |
| --- | --- |
|  | (22) |

On reducing, we reach

|  |  |
| --- | --- |
|  | (23) |

From (23), which can be achieved:

|  |  |
| --- | --- |
|  | (24) |

Where the ∙ symbol indicates a scalar product, is the magnitude of vector , is the magnitude of vector , and is the cosine of the angle formed between the and vectors. With the knowledge that expression (24) is employed to measure the similarity between vectors, this equivalence is achieved:

|  |  |
| --- | --- |
|  | (25) |

Where sim(S\_Test,S\_c ) is the similarity measurement between the and images. Thus, based on Figure 4 and Equation (25), the following conditions (as points) can be indicated:

1. If , then , because and are orthogonal, . Ergo, the and images are completely different (see Figure 5a).
2. If , then , because and have the same direction and magnitude, . For this case, the and images are identical (see Figure 5b).
3. If then ; consequently, the and images have a certain degree of similarity between them, given that the and vectors are not parallel within the texture space. Therefore, the condition is satisfied (see Figure 5c).

|  |  |
| --- | --- |
|  |  |
|  | |

**Figure 5.** (a) the and the vectors are orthogonal, and the similarity of the and equals 0; **(b)** the and vectors are parallel; hence, the and imagesare identical, and **(c)** there is a certain angle between the and the vectors; thus, the and images possess a certain degree of similarity.

Based on conditions 1-3 and on Figure 5, it is possible to measure the similarity between images within the texture space; therefore, the texture image classification is also a possibility.

**2.3. Image Classification in the Texture Space**

Figure 6 schematically displays the proposed multiclass classifier for image recognition within the texture space. The classifier consists of two phases: learning and recognition. During the learning phase, a human expert identifies and names a known image database , where each image is considered as an independent class; each class has a series of radius vectors that are calculated, and with these radius vectors, the prototype vector is obtained. This vector represents all of the local texture characteristics of the image within the class. In the recognition phase, an unknown test image is represented through a series of radius vectors and the vector is calculated with these. Afterward, the similarity between the test image and the prototype images is measured in the texture space employing Expression (25). The test image is then assigned to the most similar class; such a condition is achieved when the angle y is the smallest of these during the comparison between the and vectors (see Figure 5) and when the following condition is satisfied:

|  |  |
| --- | --- |
|  | (26) |

Ergo, the image is assigned to the class when the projection of the vector above the vector is the unit or that closest to the unit.

 **Figure 6.** A schematic representation of the multiclass classifier.

The classifier results as displayed in a confusion matrix ; the rows show the prototype images, the columns show the test images, the elements of the main diagonal correspond to the correct classification hits, and the elements outside of the diagonal represent the classification errors. The classification efficiency in terms of percentage is calculated with:

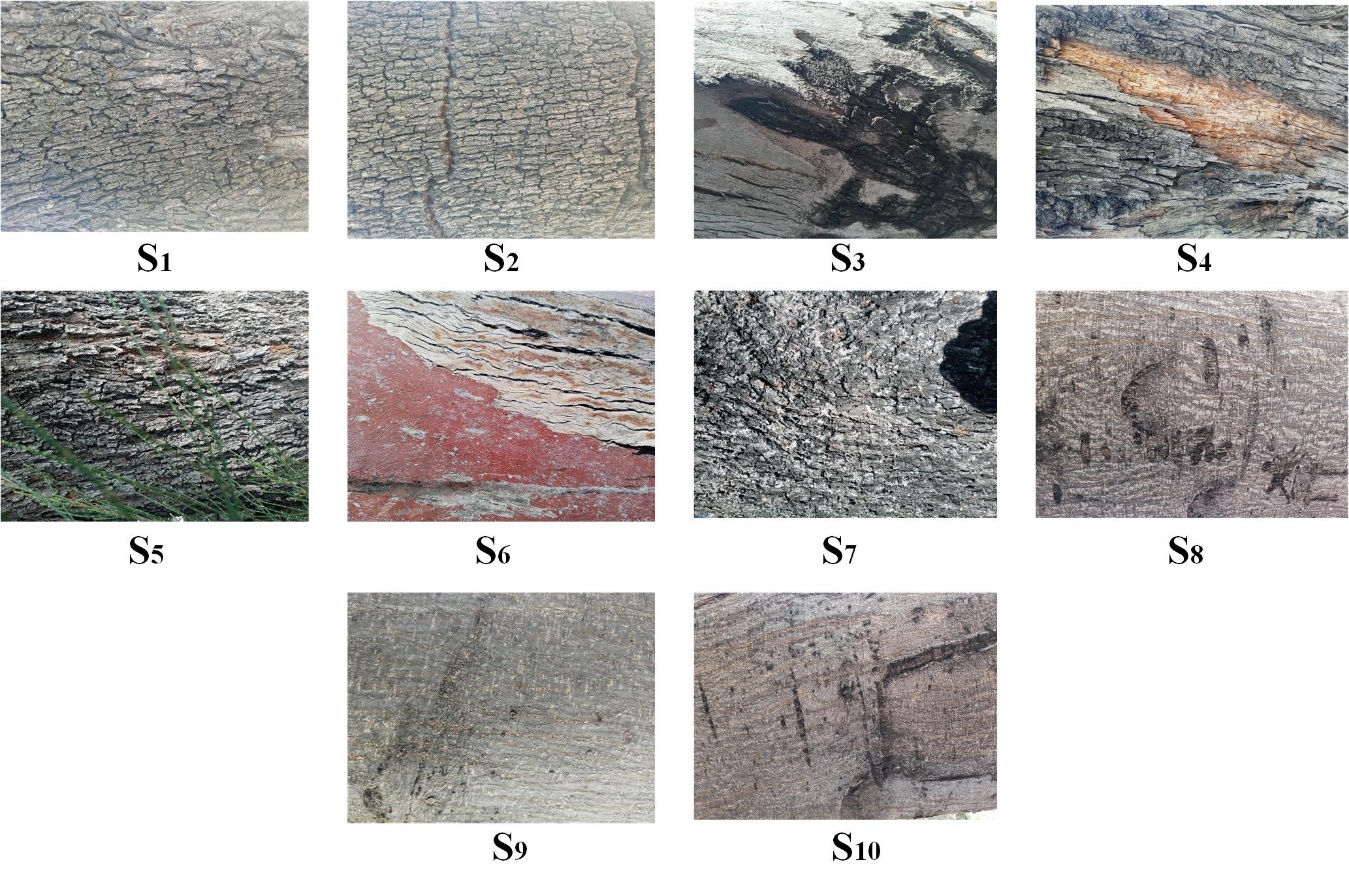
|  |  |
| --- | --- |
|  | (27) |

Where is the efficiency in terms of percentage, is the sum of all of the elements of the main diagonal in the confusion matrix, and is the sum of all of the elements within the confusion matrix.

1. Experimental Work and Results

3.1. Transformation of an Image Onto a Texture Vector

In this section, a database comprising 10 digital images of tree stems is represented through texture vectors , employing and values and an observation window of size. The database is presented in Figure 7. Each image was acquired with the Smartphone LG 50, and rotation and scale are controlled under natural illumination and with a fixed resolution of pixels.



**Figure 7.** Digital images of the tree stems employed in the experiments.

On the other hand, the transformation was performed applying the following steps: a) the RGB image acquired with the Smartphone LG 50 was transformed into a grayscale level deploying MatLab 2016b® scientific software; b) an observation window with a size is selected; c) the window is displaced element-by- element across the entire gray-level image with a size; d) for each pattern , a homogenous equation system is proposed, then its unit is calculated; e) all units are represented in the texture space as a radius vector, and f) adding together all of the radius vectors, the vector is estimated. Exercising the steps from a-f, the images in Figure 7 were represented through a texture vector . The results are presented in Table 1.

**Table 1.** Obtained texture vectors from the digital imagesshown in Figure 6.

|  |  |  |
| --- | --- | --- |
| **Transformation from image to vector** | **Vectors obtained for** | **Vectors obtained for** |
|  | **+**105**+**177608 |  |
|  | 104**+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |
|  | **+**105 **+**177608 |  |

Considering Figure 7 and Table 1, the digital image is represented in the texture space through a radius vector , whose components are dependent on the texture characteristics of the image and on the parametrization value . During the transformation, the texture characteristics of the image render the vector unique in the texture space, while the parameter operates as a scale factor.

To verify the uniqueness of each vector in Table 1, the similarity between these is measured employing the scalar product in Equation 25. The results are displayed in a confusion matrix, where the elements of the main diagonal correspond to the similarity measurements of the same vector hence, its value is the unit (marked in blue). Otherwise, the elements outside of the main diagonal correspond to the similarity measurement between two different vectors , consequently, such elements have a value lower than the unit. Table 2 and 3 present these results:

**Table 2.** Similarity measurement between vectors in the texture space, when λ=2.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Experimental results for** λ=2 (First confusion matrix) | | | | | | | | | | | |
| **Tree stems images (prototypes)** | | | | | | | | | | | |
| **Tree stems images (test)** |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1.0000 | 0.9919 | 0.9453 | 0.9997 | 0.9985 | 0.9583 | 0.9923 | 0.9915 | 0.9898 | 0.9880 |
| 2 | 0.9919 | 1.0000 | 0.9759 | 0.9916 | 0.9970 | 0.9868 | 0.9999 | 0.9999 | 0.9998 | 0.9996 |
| 3 | 0.9453 | 0.9759 | 1.0000 | 0.9424 | 0.9576 | 0.9938 | 0.9745 | 0.9764 | 0.9780 | 0.9803 |
| 4 | 0.9997 | 0.9916 | 0.9424 | 1.0000 | 0.9986 | 0.9577 | 0.9922 | 0.9912 | 0.9896 | 0.9878 |
| 5 | 0.9985 | 0.9970 | 0.9576 | 0.9986 | 1.0000 | 0.9714 | 0.9973 | 0.9968 | 0.9958 | 0.9945 |
| 6 | 0.9583 | 0.9868 | 0.9938 | 0.9577 | 0.9714 | 1.0000 | 0.9861 | 0.9872 | 0.9890 | 0.9908 |
| 7 | 0.9923 | 0.9999 | 0.9745 | 0.9922 | 0.9973 | 0.9861 | 1.0000 | 0.9999 | 0.9998 | 0.9995 |
| 8 | 0.9915 | 0.9999 | 0.9764 | 0.9912 | 0.9968 | 0.9872 | 0.9999 | 1.0000 | 0.9999 | 0.9996 |
| 9 | 0.9898 | 0.9998 | 0.9780 | 0.9896 | 0.9958 | 0.9890 | 0.9998 | 0.9999 | 1.0000 | 0.9999 |
| 10 | 0.9880 | 0.9996 | 0.9803 | 0.9878 | 0.9945 | 0.9908 | 0.9995 | 0.9996 | 0.9999 | 1.0000 |

**Table 3.** Similarity measurement between vectors in the texture space, when λ=25.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Experimental results for** λ=25 (Second confusion matrix) | | | | | | | | | | | |
| **Tree strems images (prototypes)** | | | | | | | | | | | |
| **Tree stems images (test)** |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1.0000 | 0.9919 | 0.9453 | 0.9997 | 0.9985 | 0.9583 | 0.9923 | 0.9915 | 0.9898 | 0.9880 |
| 2 | 0.9919 | 1.0000 | 0.9759 | 0.9916 | 0.9970 | 0.9868 | 0.9999 | 0.9999 | 0.9998 | 0.9996 |
| 3 | 0.9453 | 0.9759 | 1.0000 | 0.9424 | 0.9576 | 0.9938 | 0.9745 | 0.9764 | 0.9780 | 0.9803 |
| 4 | 0.9997 | 0.9916 | 0.9424 | 1.0000 | 0.9986 | 0.9577 | 0.9922 | 0.9912 | 0.9896 | 0.9878 |
| 5 | 0.9985 | 0.9970 | 0.9576 | 0.9986 | 1.0000 | 0.9714 | 0.9973 | 0.9968 | 0.9958 | 0.9945 |
| 6 | 0.9583 | 0.9868 | 0.9938 | 0.9577 | 0.9714 | 1.0000 | 0.9861 | 0.9872 | 0.9890 | 0.9908 |
| 7 | 0.9923 | 0.9999 | 0.9745 | 0.9922 | 0.9973 | 0.9861 | 1.0000 | 0.9999 | 0.9998 | 0.9995 |
| 8 | 0.9915 | 0.9999 | 0.9764 | 0.9912 | 0.9968 | 0.9872 | 0.9999 | 1.0000 | 0.9999 | 0.9996 |
| 9 | 0.9898 | 0.9998 | 0.9780 | 0.9896 | 0.9958 | 0.9890 | 0.9998 | 0.9999 | 1.0000 | 0.9999 |
| 10 | 0.9880 | 0.9996 | 0.9803 | 0.9878 | 0.9945 | 0.9908 | 0.9995 | 0.9996 | 0.9999 | 1.0000 |

Based on the results of Tables 2-3, both confusion matrixes are identical, given that the elements in their respective diagonals are the unit, and the elements outside of their diagonals are fewer than the unit. This corroborates that a digital image is represented in the texture space through a unique vector , and that the parameter , operated as a scale factor and its value, does not affect the results. Furthermore, the similarity measurement values between images above 0.94 are attributed to the parametrization of the homogenous equation system due to its resolution. This causes the third component of all of the vectors to bear the same value , and the remaining two components (first and second) are the only components scaled by the value of (see Equation 8).

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Experimental results for** λ=2 (First confusion matrix) | | | | | | | | | | | |
| **Tree strems images (prototypes)** | | | | | | | | | | | |
| **Tree stems images (test)** |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1.0000 | 0.9919 | 0.9453 | 0.9997 | 0.9985 | 0.9583 | 0.9923 | 0.9915 | 0.9898 | 0.9880 |
| 2 | 0.9919 | 1.0000 | 0.9759 | 0.9916 | 0.9970 | 0.9868 | 0.9999 | 0.9999 | 0.9998 | 0.9996 |
| 3 | 0.9453 | 0.9759 | 1.0000 | 0.9424 | 0.9576 | 0.9938 | 0.9745 | 0.9764 | 0.9780 | 0.9803 |
| 4 | 0.9997 | 0.9916 | 0.9424 | 1.0000 | 0.9986 | 0.9577 | 0.9922 | 0.9912 | 0.9896 | 0.9878 |
| 5 | 0.9985 | 0.9970 | 0.9576 | 0.9986 | 1.0000 | 0.9714 | 0.9973 | 0.9968 | 0.9958 | 0.9945 |
| 6 | 0.9583 | 0.9868 | 0.9938 | 0.9577 | 0.9714 | 1.0000 | 0.9861 | 0.9872 | 0.9890 | 0.9908 |
| 7 | 0.9923 | 0.9999 | 0.9745 | 0.9922 | 0.9973 | 0.9861 | 1.0000 | 0.9999 | 0.9998 | 0.9995 |
| 8 | 0.9915 | 0.9999 | 0.9764 | 0.9912 | 0.9968 | 0.9872 | 0.9999 | 1.0000 | 0.9999 | 0.9996 |
| 9 | 0.9898 | 0.9998 | 0.9780 | 0.9896 | 0.9958 | 0.9890 | 0.9998 | 0.9999 | 1.0000 | 0.9999 |
| 10 | 0.9880 | 0.9996 | 0.9803 | 0.9878 | 0.9945 | 0.9908 | 0.9995 | 0.9996 | 0.9999 | 1.0000 |

3.2. Image Recognition in the Texture Space

Knowing that each digital image can be represented in the texture space through a vector, the goal of this section is to prove that the digital images can be classified in the texture space. As previously presented in Figure 7, the database consists of 10 digital images of the bark of tree stems with a size of pixels; these images were acquired under natural lighting and controlled scale and rotation. The classifier employed for image recognition was described in Section 4. In both phases, the same images are employed for both learning and recognition, along with the same observation window size of pixels. The similarity measurement in the texture space is performed considering the maximal likeness between the and vectors (Equations 25 and 26). To conclude, the classification results are presented through two confusion matrixes: Table 4 displays the confusion matrix for and Table 5 presents the confusion matrix for .

**Table 4.** Confusion matrix obtained for the image classification when .

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Experimental results for** λ=2 | | | | | | | | | | | |
| **Tree stems images (prototypes)** | | | | | | | | | | | |
| **Tree stems images (test)** |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

**Table 5.** Confusion matrix obtained for the image classification when

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Experimental results for** λ=25 | | | | | | | | | | | |
| **Tree stems images (prototypes)** | | | | | | | | | | | |
| **Tree stems images (test)** |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

It is worth recalling that the elements of the main diagonal in these matrixes represent the correct classification hits, and the elements outside of the main diagonal are the identification errors. In this manner, based on Equation (27) and Tables 4 and 5, the classification is:

|  |  |
| --- | --- |
|  | (28) |

Where is the image classification efficiency in terms of percentage for and is the image classification efficiency for . The efficiency is 100% in both cases. This further confirms that the proposed transformation in Section 2, along with the classifier described in Section 4, entertain a high efficiency and that the recognition of the images can be performed in the texture space. The high efficiency is attributed to the following points:

1. In the transformation, the image is completely characterized through its local texture characteristics, and these are represented by the texture vector .
2. The digital image is essentially a field of randomness, given the nature of the light source and the noise detected by the system; henceforth, for each image , a unique vector is generated in the texture space with a particular direction and magnitude that differ for each class.

Nonetheless, the efficiency of our proposal can be reduced if the digital images are classified dynamically (in real time). This is due to the fluctuation of the light source temporarily and spatially. Consequently, for each instant of time, the pixels of the digital camera vary in intensity. In other words, the noise during the acquisition of the image increases; thus, the texture vector  changes, causing recognition errors.

1. Discussion

In this paper, the transformation is proposed where is a grayscale image and is a vector in a new space, which is denominated texture space. Essentially, the transformation consists of representing the image through a series of radius vectors in the texture space, with each radius vector a texture unit , and this is calculated by solving a homogeneous equation system. Afterward, the vector is calculated by the sum of all of the radius vectors and, subsequently, all of the local texture characteristics of the image-under-study are considered in it. Its direction and magnitude are in agreement with the randomness of the digital image and, for each image , a unique vector is generated. Additionally, a multiclass classifier is proposed and applied within the texture space. where the vector is employed as a characteristics vector, demonstrating its potential application for image classification. Based on these results, the following points are worth mentioning:

1. The image is fully characterized in the transformation , where the texture space is represented by the texture vector . The new transformation can be termed Vectorial Image Representation on the Texture Space (VIR-TS) because, in the image transformation, the image comes to be represented by the vector .
2. Due to the irregular nature of the light source and the noise during the photodetection process, the image is considered a field of randomness; consequently, a unique vector is generated for each digital image (see Table 1).
3. The vector withholds all local texture characteristics of the image-under-study, given that the vector is calculated by the sum of all of the radius vectors, where a radius vector is defined as texture unit .
4. The texture unit possessed a vectorial character because it is calculated by solving a homogeneous equation system of .
5. The texture vector can be employed as a characteristics vector in classifiers with a high efficiency (see Tables 3 and 4).
6. The value employed for the solution of the homogeneous equation system does not affect the results of the image recognition.
7. The transformation has a potential application in the development of artificial vision systems focused on the recognition of digital images.
8. In the experimental work, the number of classes does not affect the results of the classification efficiency, given that each digital image is represented by its own vector in the texture space (see point 2).
9. Because medical images contain local textural features which can be extracted through local analysis [3,4,26,27], and knowing that the technique reported in this work also extracts texture features based on local analysis local, then the VIR-TS transform and the classifier described in section 2.3 can be applied in medical image recognition. The benefit would be the development of medical diagnostic systems with high efficiency, easy to implement because the definition of the texture unit is based on a linear transformation and not on the encoding of a pattern [21,28], where the overflow of physical memory of the computer is possible [29].
10. Comparing the statistical texture extraction techniques reported in reference [21] with the VIR-TS technique based on linear transformations, both texture extraction techniques are very different. In statistical techniques, the texture unit is calculated based on the encoding of discrete random patterns located on the digital image, its texture unit is considered a random event and the texture characteristics are represented through a discrete histogram. While, in our technique called VIR-TS, the texture unit is calculated based on a linear transformation, its texture unit is a radius vector and the texture features are represented in a texture space through a random vector.

The Vector Image Representation on the Texture Space (VIR-TS) transformation is very different from the statistical techniques reported in references [21]. In the VIR-TS transformation, the texture unit is a radius vector, the vector is calculated by solving a homogeneous system of equations and its graph can be visualized in the texture space. With the transformation, the digital image *S* is expressed in texture space by the random vector , which is made up of three components and whose addresses are . Because the image is vector-represented, image classification in texture space is done by measuring the similarity between the prototype vectors and the test vector. Their similarity is calculated through the projection between both vectors. Finally, the test image is assigned to the most similar class. Based on the experimental work, the VIR-TS transformation has high classification efficiency because its texture feature extraction efficiency is very high. Furthermore, its implementation is very easy because the digital image is represented through a three-component random vector.

With the knowledge that our proposal has a potential application in image recognition, our future lines of research will include the following: rendering the VIR-TS transform invariant to rotation and scale; proposing the VIR-TS transform for color image classification; applying the VIR-TS transform in the recognition of biomedical images, and performing an efficiency study of classification in images with noise.

5. Conclusions

In this paper, the Vectorial Image Representation on the Texture Space (VIR-TS) transform is proposed and applied. The VIR-TS transform is based on the extraction of the local texture characters of the image S and represents these through the vector C ⃗ in the texture space. Each radius vector is a texture unit T ⃗, which is estimated by solving a homogeneous equation system of 3×3. In the texture space, each image has a corresponding unique vector, given that the image is a random field of pixels. Experimentally, the vector C ⃗ was employed as characteristics vector in a new multiclass classifier; thus, the high efficiency of the VIR-TS transform was corroborated through the classification of the digital images of the stem of trees. The efficiency reached 100%; however, in applications under natural environments, its efficiency may reduce significantly due to the noise in photodetections and the random nature of light.

The VIR-TS transform has a potential application for the localization of missing persons and for the classification of medical images.

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